Propulsion Application of the Modified Penning Arc Plasma Ejector

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Several critical problems, which arise from the adaptation of a modified Penning-type arc to plasma propulsion, are examined. The use of a cold cathode discharge using an alkali metal fuel is considered as an attractive alternative to thermionic discharges, particularly where plasma exhaust velocities are characterized by $2000 < I_{sp} < 4000$ sec. Analyses show that the cathode losses are dependent upon the probability of ionization, secondary electron yield, sputtering, and exhaust efficiency. Particular emphasis has been placed upon the problem of ejecting the exhaust plasma that is formed in the arc, first through a divergent solenoidal magnetic field and then into free space. Experimental evidence, using a special cold-cathode plasma generator, has confirmed the calculations of the critical problems to an order of magnitude. Both experimentally and theoretically, within the limits of the modified Penning plasma generator, the results indicate that at present it is inefficient for propulsion and that there is an especially important associated problem of plasma ejection through the magnetic gradient exhaust region.

Nomenclature

P = mv, the electron momentum, and v is electron velocity
ΔP = transverse momentum acquired by a scattered electron
m = electron mass
e = electronic charge
n = number of ions (electrons) per unit volume
k = Boltzmann's constant
T = electron temperature, °K
λ_D = Debye shielding distance

I. Introduction

A MODIFIED Penning arc, which can eject a plasma with a range of specific impulse between that of arc jets and contact ionization motors, has been considered as a possible electrical propulsion device. Several critical problems associated with such a device have been examined from both a theoretical and experimental point of view, and some of the results obtained are presented in this paper.

Briefly, this type of plasma generator consists of three electrodes: a cylindrical anode, a hollow cathode, and a closed cathode. These electrodes provide a potential trough, which, in conjunction with a coaxial solenoidal magnetic field, embodies the principle of a Penning discharge; the magnetic field confines the motion of electrons escaping from the cathode in axial directions and causes some of them to oscillate within the tube in the electrostatic potential well created by the anode and the cathodes at each end of the tube until they diffuse to the anode through collisions. The magnetic field is of the order of 1000 gauss or less, and the oscillating electrons with an initial energy of the order of a few hundred volts are able to create ions in the neutral gas by inelastic collisions until their energy is spent.

In a true Penning discharge, the electrons leaving the closed cathode are secondary electrons resulting from ions that originate in the plasma region (trough area) and strike the cathode. The kinetic energy of the ions upon arrival at the cathode is fairly high and is obtained from acceleration across the plasma sheath formed near the closed cathode. Because the secondary electrons caused by ion bombardment are the predominant source of electrons, and because the secondary electron coefficient increases essentially linearly with incident ion energy over the applicable range of up to several kilovolts, the discharge voltages for cold cathode operation are generally greater than for a corresponding hot filament Finkelstein-type discharge.[1]

Meyerand et al.[2, 3] have investigated an electric field configuration of an open cathode oscillating electron discharge using a hot filament electron emitter with several gaseous fuels for propulsion purposes. However, it appears that a cold-cathode discharge (more closely related to the original Penning-type discharge[5]) may be more advantageous under certain conditions; therefore, some exploratory work was undertaken.

In order to identify and investigate the critical problem areas associated with this cold-cathode type of electric propulsion device, it has been necessary to examine in detail some of the aspects of its operation. There are at least four such important aspects of operation which must be considered. First among these is the mechanism of the acceleration of the plasma. It has been assumed that in the arc there exists an adequate acceleration mechanism that provides a suitable exhaust velocity, and that the potential gradient mechanisms are related, at least in principle, to the distributed ion production in the arc (with subsequent nonthermal equilibrium electron distribution), as has been examined by Meyerand, Davis, et al.[2–4] This problem is not considered further in the present paper, but three remaining operational aspects are discussed.

The choice of propellant and its method of injection is a second aspect of the device operation which was considered. The use of alkali metals as a fuel is attractive, particularly in the range of $2000 < I_{sp} < 4000$, because of the low ionization potentials, low work functions, huge collision cross sections,[6] and large gap between the first and second ionization potentials (30 to 40 v for Na, K, and Rb). Accordingly, an experimental plasma generator using cesium was investigated, and some results, including a comparison with argon operation, are described in a later section. By using a cold-cathode discharge, it is possible to inject the fuel in a controlled way through the closed cathode, thus taking advantage of a unique feature of neutral gas injection providing a local high gas density around the jet. In this manner, it is possible to achieve a fair probability of gas ionization located at a potential removed from the closed cathode and yet coupled

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with a significant probability that electrons and ions will escape the vicinity toward the exhaust region.

Considerations of the cathode operation provide the third general aspect that was examined. In the different cases of cold and hot cathode operation (secondary vs thermionic electron emission), the choice of cathode materials is influenced by sputtering and cathode power requirements. In both cases, but for different reasons, the use of a low work function surface seems advantageous. For the hot cathode, use of a low work function surface reduces chiefly the heater input power. For the cold cathode, however, since the operating voltages tend to be higher than for the hot cathode case, it is possible to reduce the operating voltage by introducing an alkali-metal fuel, thus lowering simultaneously the operating voltage, the cathode power loss, and the cathode sputtering rate.

The ejection of the plasma from the magnetic field region of the discharge is the fourth aspect of the device operation. The electrons in the ejected plasma tend to follow the divergent magnetic lines of force, and thus the exhaust plasma tends to blow up in such a way as to reduce the net thrust substantially if the fields are large or if the current density is small. The coulomb force between electrons and ions (which are relatively unaffected by the magnetic field due to their large momenta) and an axial electric field (which can exist between the arc and a laboratory beam collector) tends to reduce the exhaust blowup. This problem has been analyzed and the results discussed as a function of magnetic field, plasma density, and exhaust electron temperature.

The results of the present investigation of the Penning arc, modified for plasma ejection, are that it is an interesting device for electric propulsion, but at present it is both experimentally and theoretically inefficient. The reason for this is probably because very little is known about the processes involved and how to optimize them. There appear to be significant advantages in the use of alkali metals as a fuel, particularly when used with a cold-cathode discharge of the Penning type, where generally the effect is to achieve lower operating voltages as well as to reduce cathode sputtering. Among the several areas outlined in the foregoing, the problem of plasma ejection especially through the magnetic gradient region appears to warrant further experimental work, particularly since it relates to the general problems of fluid plasma flow of other plasma devices as well.

II. Beam Ejection from the Magnetic Field

In an electrically neutral plasma generated with a directed energy equivalent to a few hundred electron volts per particle, the question arises as to whether this plasma can leave the high magnetic field region and escape without serious perturbation into free space. Under the assumption that the plasma is electrically neutral with finite electrical conductivity, such a conducting gas when ejected will experience forces due to the gradient of the magnetic flux density, typically of the order of 100 gauss/cm. The exact behavior of the plasma beam may be enormously complicated, but with some simplifying assumptions the behavior of the plasma may be estimated. Although the effect would depend upon the electron velocity distribution and mean free path in the escaping plasma, a simple picture would be that the electrons tend to follow the field lines of the solenoid. This would pull the plasma stream apart as it passed through the diverging magnetic field, since the ions must follow the electrons if the net charge density is to remain small.

The plasma in the arc is not in thermal equilibrium in the classical sense. However, if one adopts the simplifying assumption that the electrons accompanying the plasma have an isotropic velocity distribution in the ion center of momentum reference frame, then the plasma beam expands as it passes through the changing magnetic transition region. The degree of expansion may be predicted approximately by assuming a Maxwellian distribution. The actual distribution may give somewhat different results, but since the behavior investigated below depends more on the average electron energy than on the details of the velocity distribution, the Maxwellian distribution assumption probably gives an acceptable estimate. It is assumed that the electron mean free path is small compared to the plasma dimensions, and that the average electron velocity is much larger than the ion velocity.

To calculate the mean free path μ of electrons, employ the multiple coulomb scattering theory,[7] and take as the "mean free path" that distance in which the average rms momentum transfer to a moving electron is equal to its initial momentum. This expression can be found from

$$d(\Delta P)^2/dt = (8\pi n\lambda_0^2 P^3/m) \log Y \quad (1)$$

where

$$Y = \lambda_D/\lambda_0 = (1/\lambda_0)(kT/4\pi ne^2) \quad (2)$$

and

$$\lambda_0 = e^2/mv^2 \qquad \lambda_D = (kT/4\pi ne^2)^{1/2} = 6.90(T/n)^{1/2}\text{ cm} \quad (3)$$

Equation (1) applies to a fully ionized plasma with singly charged ions.[7]

To find the "mean free path," note that the time τ required for a transfer of momentum equal to the original momentum is given by

$$\tau = P^2 dt/d(\Delta P)^2 \quad (4)$$

and the "mean free path" μ by

$$\mu = \tau v = 1/8\pi n\lambda_0^2 \log Y = (mv^2)^2/8\pi e^4 n \log Y \quad (5)$$

For a given electron distribution (e.g., Maxwellian), the foregoing expression properly should be averaged over the velocity distribution itself. However, for estimate purposes, $3kT$ is substituted for mv^2, which corresponds to the average energy in a Maxwellian distribution. Thus,

$$\mu \sim 9(kT)^2/8\pi\, e^4 n \log Y \quad (6)$$

For a typical case of 10^{13} ions/cm^3 ($n \approx 2 \times 10^{13}$/cm^3, since the scattering contribution of the electrons is roughly equal to that of the ions),

$$\mu \sim 4 \times 10^{-4}\text{ cm} \qquad T \sim 10^3\ ^\circ\text{K}$$

$$\mu \sim 3.5 \times 10^{-2}\text{ cm} \qquad T \sim 10^4\ ^\circ\text{K}$$

and the assumption of short mean free paths is justifiable. If, however, $T \sim 10^5$ (i.e., 10 v electrons), then $\mu \sim 3$ cm and the assumption is questionable.

Also considered is the plasma electrical conductivity in a strong magnetic field, where the ratio of transverse to longitudinal conductivity is about a factor[7] of 3. A "strong" magnetic field in this case means that the radius of gyration is much smaller than the collision mean free path, where the radius of gyration r_c is given by

$$r_c = 3.37(V)^{1/2}/B\text{ cm} \quad (7)$$

with V the particle energy in electron volts and B the magnetic field in gauss. For one volt electron (i.e., $T \sim 10^4$) and $B \sim 300$ gauss, $r_c \sim 0.01$ cm, which is less than the "mean free path" of 0.035 cm in the foregoing. However, if $T \sim 10^3$ (i.e., 0.1 v), then $r_c \sim 0.003$ cm, which is greater than the mean free path of 0.0004 cm in the foregoing. Thus, for the lower temperature, isotropic plasma conductivity would be used, but the conductivity transverse to the magnetic field would be reduced about a factor of 3 relative to the longitudinal conductivity at the higher temperature.

In order to estimate the effect on the plasma when it is ejected from the magnetic field, it is assumed that the plasma is fully ionized, that the electrons are sufficiently close to thermal equilibrium that the plasma conductivity equations apply, and that the plasma is nearly electrically neutral and reasonably uniform. As a first approximation, then, the plasma is treated as a conducting fluid as it is ejected from the region of the uniform, time independent, axial magnetic field into the field-free exhaust as shown in Fig. 1.

From Maxwell's equations

$$\nabla \times \mathbf{E} = -(\partial \mathbf{B}/\partial t) \tag{8}$$

and using the definition of electrical conductivity, $\mathbf{j} = \sigma \mathbf{E}$, where $\mathbf{j}$ is the current density in the plasma and σ the conductivity. Then

$$\nabla \times \mathbf{j} = -\sigma\, \partial \mathbf{B}/\partial t \tag{9}$$

and σ is a scalar constant.§ The Lorentz force on the plasma is given by $\mathbf{a} = \mathbf{f}/\rho = \mathbf{j} \times \mathbf{B}/\rho$, where ρ is the plasma density.

Attention is now fixed on the radial acceleration of the plasma, and the first-order assumption will be made that the plasma expansion is small. Then if v_z is the plasma velocity in the z direction, one may replace $(\partial \mathbf{B}/\partial t)$ with $(\partial \mathbf{B}/\partial z)v_z$, where the magnetic field is constant with time. Then, one finds, integrating over a disk of radius r,

$$\int_{\text{area}} (\nabla \times \mathbf{j})\cdot(\mathbf{z}/z)\, dA = 2\pi r j_\varphi = -\sigma \int v_z\, (\partial \mathbf{B}/\partial z)\cdot(\mathbf{z}/z)\, dA = -\sigma(\partial \varphi_z/\partial z)\, v_z \tag{10}$$

where the azimuthal component of j_φ is independent of azimuth by symmetry, and φ_z is the total magnetic flux through the disk of radius r. Then the radial acceleration of the plasma is given by

$$a_r = j_\varphi B_z/\rho = -(\sigma v_z B_z/2\pi r\rho)(\partial \varphi_z/\partial z) \tag{11}$$

Now, under the assumption that the plasma expansion is small, one can calculate the final divergence angle of a portion of the plasma at the radius r by

$$\theta = \frac{v_r}{v_z} = \frac{1}{v_z}\int_0^\infty a_r dt \approx \int_0^\infty \frac{a_r}{(v_z)^2}\, dz = -\int_0^\infty \frac{\sigma B_z}{2\pi r \rho v_z}\,\frac{\partial \varphi_z}{\partial z}\, dz \approx -\int_0^\infty \frac{r\sigma B_z}{2\pi \rho_0 R^2 v_z}\,\frac{\partial \varphi_z}{\partial z}\, dz \tag{12}$$

where $z = 0$ is taken within the region of constant magnetic field, ρ_0 is the plasma density in this region, and R is the initial radius of the plasma. This integral is evaluated approximately for the case of small plasma expansion by assuming that r changes very little and σ remains nearly constant. Note that this underestimates the plasma divergence angle, since if there is a significant divergence, r must increase during the expulsion, consequently increasing θ, as can be seen from the preceding expression. Then approximate $\varphi_z \approx B_z \pi R^2$, and finally

$$\theta \sim -\int_0^\infty \frac{\sigma B_z R}{2\rho_0 v_z}\,\frac{\partial B_z}{\partial z}\, dz = \frac{\sigma B_0^2 R}{4\rho_0 v_z} \tag{13}$$

where B_0 is the initial field in the uniform field region. Observing that

$$\rho v_z \approx nMv_z = (J/e)M \tag{14}$$

where M is the mass of an ion and J is the "current density" of the ions, i.e., the current density that would be measured if only the ionic component of the streaming plasma were present, one finds

$$\theta \sim (\sigma B_0^2 R/4J)(e/M) \tag{15}$$

§ Actually, a plasma in a magnetic field is being considered, so that σ is a tensor, as noted earlier. However, the effect of this upon subsequent calculations is negligible, as one need only consider that component of the current flowing transverse to the magnetic field. Thus, one may treat σ as a scalar, providing one uses the proper "transverse" value for calculation.

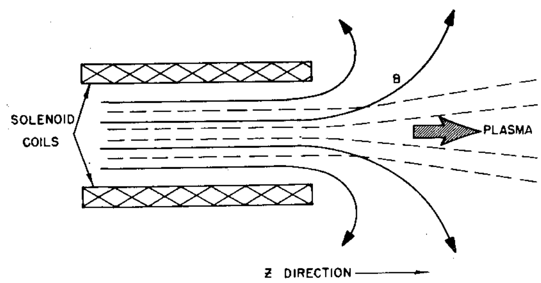

Fig. 1 Plasma ejection in a divergent magnetic field

The preceding expression is only approximate, of course, but it is nearly correct for a small expansion of the plasma. As has been noted, it underestimates the effect if the expansion is actually severe, so that if a large number compared to unity is found for θ, it simply indicates that the plasma follows the magnetic field lines rather closely.

The number now needed for the foregoing expression is the plasma electrical conductivity σ. This is given in Ref. 7 for singly ionized species as

$$\sigma = \frac{\lambda_E 2(2kT)^{3/2}}{\pi^{3/2} m^{1/2} e^2 c^2 \log Y} = \frac{1.53 \times 10^{-4} T^{3/2}}{\log Y}\ (\text{ohm-cm})^{-1} \tag{16}$$

where the units are emu and $\lambda_E = 0.582$.

Using the numbers given by the foregoing expression, one now may compute θ. Table 1 gives the results for a singly ionized cesium plasma, which is an exceptionally favorable case because of the small value of e/M. The results are given in terms of the angular spread of the outer edge of a plasma beam of uniform density, containing the current noted in the table. These results are derived from Eq. (15), where I is the total current (of the ionic component of the plasma) contained within the initial beam radius R. For purposes of computing σ, the energy of particles in the beam is taken to be 100 ev. This assumption is not at all critical, as σ is very insensitive to energy. The voltage could be taken from 10 to 1000 v with very little alteration of the results.

Some features of Table 1 should be noted. For the $T = 10^4$ °K columns, and for both temperatures at the $B = 5000$ gauss field, the conductivity was reduced arbitrarily to the appropriate value transverse to a strong magnetic field, as explained earlier. This value is found by reducing the number obtained from the earlier conductivity expression by a factor of $3\pi/32 \approx 0.3$.

It is, of course, obvious that the numbers in Table 1 for $\theta \gg 1$ are completely meaningless in a quantitative sense, as the nature of the approximation used in calculating them does not permit this. However, these large numbers are a certain indication that the plasma follows the actual diverging field lines very closely, and that the shape of the existing exhaust is very nearly that of the field lines. When $\theta \ll 1$, the calculation is quantitatively correct, assuming the validity of the basic assumptions.

The most striking thing about these numbers is the indication that very small diameter beams and high currents are necessary in order for the plasma to escape. It would be very surprising if the electrons in the exhaust had a temperature as low as 10^3 °K, and probably the larger value of 10^4 °K is more appropriate. Then, with the more typical larger magnetic field, it can be seen that comparatively high currents ($\sim$10 amps) are necessary even for a beam as small as 1 cm in radius to leave the magnetic field. It would seem impossible on the basis of this simple calculation for a beam as large as 10 cm in radius to escape the magnetic field even with currents as high as 1000 amps except in the case of the smaller magnetic field. The only cases that look particularly favorable are the very small ($\sim$ a few millimeters) radius, high current beams. Whether or not these can be produced without the use of a very strong confining mag-

Table 1 Beam divergence dependent upon magnetic field, current, and diameter (cesium plasma)

Beam radius, R, cm	Beam current, I, amp	θ (divergence angle), rad					
		$B_0 = 200$ g		$B_0 = 1000$ g		$B_0 = 5000$ g	
		$T = 10^3$ °K	$T = 10^4$ °K	$T = 10^3$ °K	$T = 10^4$ °K	$T = 10^3$ °K	$T = 10^4$ °K
0.1	0.1	2.42×10^{-4}	0.128×10^{-2}	0.605×10^{-2}	3.20×10^{-2}	4.46×10^{-2}	0.80
0.1	1.0	3.22×10^{-5}	1.49×10^{-4}	0.805×10^{-3}	0.373×10^{-2}	5.94×10^{-3}	9.33×10^{-2}
0.1	10.0	4.84×10^{-6}	1.80×10^{-5}	1.21×10^{-4}	4.5×10^{-4}	8.93×10^{-4}	1.12×10^{-2}
1.0	0.1	0.16	0.995	4.00	24.9	29.5	6.23×10^{2}
1.0	1.0	1.93×10^{-2}	0.112	0.483	2.80	3.57	70.0
1.0	10.0	0.242×10^{-2}	1.28×10^{-2}	6.05×10^{-2}	0.320	0.446	8.0
1.0	100.0	3.22×10^{-4}	0.149×10^{-2}	0.805×10^{-2}	3.73×10^{-2}	5.94×10^{-2}	0.933
10.0	1.0	13.8	89.3	3.45×10^{2}	2.23×10^{3}	2.55×10^{3}	5.58×10^{4}
10.0	10.0	1.60	9.95	40.0	2.49×10^{2}	2.95×10^{2}	6.23×10^{3}
10.0	100.0	0.193	1.12	4.83	28.0	35.7	7.00×10^{2}
10.0	1000.0	2.42×10^{-2}	0.128	0.605	3.20	4.46	80.0

netic field is not immediately obvious. The strong field, of course, is undesirable, as can be seen from the B^2 dependence of the divergence angle. It also is to be noted that the lighter atomic species are less favorable, since the divergence angle increases approximately as the inverse ratio of the atomic mass, other things being equal.

These considerations are of paramount importance with regard to the use of such a device as a propulsion source. Clearly, a propulsion device is useless if it cannot produce a directed beam, and no thrust is to be expected if the exhaust plasma follows the magnetic field lines out of the exhaust back to the front of the discharge tube. The foregoing calculations are certainly of the simplest type that can be applied in a basically complex plasma situation, and it is reasonable to question their validity. The quasi-equilibrium assumptions may be seriously in error, and the actual plasma configuration could be much more complicated. However, it is possible that the situation as described in the calculations is related to what must happen in free space when the device is used as a propulsion method, but quite different results could be expected in the laboratory.

For example, one may imagine an admittedly crude model in which the electron component of the outgoing plasma has no net velocity with respect to the sources. That is, the electrons form a (thermally excited) cloud that is on the average stationary with respect to the vehicle, but whose density is adequate to cancel the ion space charge. Then, since the electrical conductivity associated with the ion component is negligible, the ions would be allowed to pass through the diverging magnetic field with no appreciable effect. The net current to a laboratory collector could still be zero, as a small, high energy stream of electrons would be able to pass down the center of the ion beam without serious divergence, thus canceling the net current of the ion beam.

However, this situation requires an axial electric field of a calculable value. This can be seen most simply if one views the situation in the ion center-of-momentum system. Here the electron cloud appears to have a net velocity and appears as a current directed toward the emitter. The magnitude of the current is just that of the escaping ion beam, and the magnitude of the electric field required to maintain the current is calculable from the current density and the plasma electron conductivity, $\mathbf{E} = \mathbf{J}/\sigma$. For a cesium ion current of about 1 amp/cm^2 and electron cloud temperature of 10^4 °K, the required field is about 0.06 v/cm. If the cloud temperature is 10^3 °K, the required potential is about 1 v/cm. For larger currents the effect becomes more marked. At 10 amp/cm^2, the 10^4 °K temperature would indicate an axial field of about 0.5 v/cm, whereas the lower temperature would yield a field of about 8 v/cm. In the laboratory, with a collector not too distant from the emitter, such a situation would be quite acceptable, as the small potential gradient would not be a problem, or even noticeable, and the ion stream would emerge without divergence from the magnetic field. It is, in fact, interesting to note that in a laboratory situation, just where the first analysis would indicate serious beam divergence (i.e., with low current density), the other mechanism would produce good results, with only a very small axial field.

Whether such an expulsion mechanism as just described, even if physically possible, would operate in free space is open to question. The axial field described is in a direction such as to retard the small high energy electron stream. The field gradient would not terminate until this stream scattered into the general cloud, eventually forming a uniform plasma. From the manner in which it was derived, it is clear that this voltage gradient and the current passing along it (either the ionic or electron cloud, depending on the frame of reference) simply produce ohmic heating of the electron cloud and ions and do not change the directed energy of the ions. The energy for this heating comes from the fast electron stream. The heating would result in higher electron temperatures, enhanced conductivity, and thus lowered potential gradient. This lower gradient in turn would give the fast electron stream a greater distance in which to scatter, and so on. It is possible that this could evolve into a stable situation, but its complexity, along with the rather strong energy dependence of important parameters such as electrical conductivity and electron mean collision path, makes any realistic quantitative calculation based on this model problematical. The foregoing picture is qualitative, with emphasis on simplification.

III. Emitter-Cathode Considerations

In general, the power loss to the cathode (apart from power to maintain the closed cathode at operating temperature) is due to ion bombardment. It is assumed here that the power to the open cathode is negligible and that the output of the power supply is apportioned only between the arc and the exhaust.

Under these assumptions the cathode current is essentially equal to the arc current and consists of a sum of the electron current leaving the cathode and the ion current arising from within the plasma. It is convenient to relate the currents to the equivalent ion current i_+ exhausted by the device.

If i_e is the electron current leaving the cathode, and if η_i is the number of ion-electron pairs created by an emitted electron, then

$$\eta_i = (i_+)_{\text{total}}/i_e \tag{17}$$

and

$$i_+ = (1 - \alpha)(i_+)_{\text{total}} = (1 - \alpha)\eta_i i_e \tag{18}$$

In this expression, α is the fraction of ions that return to the closed cathode and $(1 - \alpha)$ is ejected into the exhaust. The arc current i_{arc} is then

$$i_{\text{arc}} = [(1 + \alpha\eta_i)/(1 - \alpha)\eta_i]i_+ \tag{19}$$

If the normalized energy distribution of the ions that arrive at the cathode is given by the function $g_a(y)$, then the average dimensionless energy is

$$\bar{y}_a = \int_0^1 y\, g_a(y)\, dy \tag{20}$$

where $y = (v/v_0)^2$, v is the ion velocity, $v_0 = (2eV/M)^{1/2}$, V is the arc voltage, and M is the ion particle mass.

In general, the electrons emitted from the cathode are produced either by thermionic or by secondary emission processes, and usually both sources will play a partial role. The secondary electron coefficient δ will depend upon both the cathode material and the bombarding ion energy, and coefficients are typically of the order of unity to 0.1 under the conditions encountered here.[8] For cold cathode operation, each of the $\alpha\eta_i$ ions that return to the cathode produce δ electrons, so that a continuous discharge can be maintained if

$$\delta\alpha\eta_i = 1 \tag{21}$$

Consider two examples, a first case where $\alpha = 0.2$, $\eta_i = 0.7$, and a second case where $\alpha = 0.6$, $\eta_i = 5.0$. Thus $\delta\alpha\eta_i = 0.028$ and 0.6, respectively, and the discharge in the first case must be assisted by thermal emission, whereas very little thermal emission is required for the latter.

In order to estimate the cathode temperature, Richardson's equation may be applied because of emission limiting circumstances. For a cathode temperature T_c, effective work function φ_{eff}, and an emitting area that is a fraction r_c of the beam area A, then

$$J_e = 120\, T_c^2 \exp\left(-\frac{e\varphi_{\text{eff}}}{kT_c}\right) = \left(\frac{1 - \delta\alpha\eta_i}{r_cA}\right) i_e = \frac{4i_+}{r_cA}\left[\frac{1 - \delta\alpha\eta_i}{(1 - \alpha)\eta_i}\right] = \left(\frac{4i_+}{r_cA}\right)(\xi) \tag{22}$$

For the preceding examples $\xi_1 = 1.73$ and $\xi_2 = 0.20$. Since $e\varphi_{\text{eff}} \gg k\,T_c$, then

$$(kT_2/kT_1) \cong [1 + (kT_1/e\varphi_{\text{eff}})][\ln(\xi_2/\xi_1)]^{-1} \tag{23}$$

where T_2 and T_1 are values of T_c in the thermionic and cold emission cases, respectively. Typically representative results are $900 < T_2 < 1300°C$ and $400 < T_1 < 500°C$ if $\varphi_{\text{eff}} = 2.0$ v for cesium.

The power P_{cath} dissipated at the cathode of course regulates T_c for cold cathode operation. If the ion current density to the cathode were increased, it is possible under some circumstances that the cold cathode would become a thermal emitter predominantly, and an unstable runaway electron emission would prevail. The cathode power dissipation therefore is critical to operation and

$$P_{\text{cath}} = V\alpha(i_+)_{\text{total}}\,\bar{y}_a = Vi_{\text{arc}}\,[\eta_i\alpha/(1 + \alpha\eta_i)]\bar{y}_a \tag{24}$$

Whereas P_{cath} can be dissipated by either thermal conduction or radiation from the cathode and thus establish a cathode temperature T_c, there is the additional feature that P_{cath} is essentially a loss as far as power efficiency for propulsion is concerned.

The ratio η_{ea} of exhaust power to arc power is

$$\eta_{ea} = \frac{P_{ex}}{P_{\text{arc}}} = \left[\frac{\eta_i(1 - \alpha)}{(1 + \alpha\eta_i)}\right](\bar{y}_e + \bar{y}_{e-}) \tag{25}$$

where $\bar{y}_e$ and $\bar{y}_{e-}$ are the average energies of the exhaust ions and electrons, respectively. If the electrons and ions have similar velocities in the plasma stream, then $\bar{y}_{e-} \ll \bar{y}_e$, and $\bar{y}_{e-}$ becomes relatively insignificant. The power efficiency η_p also is expressed easily in terms of η_{ea}:

$$\eta_p = \eta_{ea}/(1 + \eta_{ea}) \tag{26}$$

For a numerical example, the authors of this paper have assumed an idealized triangular energy distribution for the exhaust ions, so that $\int_0^1 yg_e(y)dy = 0.598$. Thus the assumption leads to a power efficiency of 0.33 and $\eta_{ea} = 0.5$. For the thermally assisted arc, value for α and η_i just given result in an ion cathode current of 0.25 i_+ and electron emission of 1.8 i_+. For the cold cathode values, the cathode ion current increases to 1.5 i_+, whereas the electron contribution decreases to 0.5 i_+. Under these conditions, the total cathode current (ion + electron component) remains relatively unchanged at 2.0 i_+ for either case.

Choice of the cathode material of course is affected by thermionic properties, but, in addition, sputtering damage due to ion bombardment must be considered. The ratio of incident ions to sputtered cathode particles, averaged over the ion energy, gives an average sputtering rate $\bar{S}$ that yields

$$\dot{M}_c = (\bar{S}\,\overline{M}_c/e)[\alpha/(1 - \alpha)]i_+ \tag{27}$$

where $\overline{M}_c$ is the average mass of a sputtered atom or molecule. Because there exists, in general, a threshold energy of the order of 50 ev below which $\bar{S}$ effectively vanishes, there is an advantage in keeping the arc potential and average energy of the ions as close as possible to this threshold or below.

The sputtering ratio for singly charged argon ions of about 400-v energy, for example, are typically of the order of 1 to 2, depending upon the cathode material, and the ratio drops an order of magnitude for energies of the order of 100 to 150 v. The sputtering rate of the cathode material, related to the ejected flux by

$$\dot{M}_c/\dot{M}_e = [\alpha/(1 - \alpha)](\overline{M}_c/M)\bar{S} \tag{28}$$

where $\dot{M}_e$ is the expellant mass flux of atomic mass M, must be very small compared to unity (and, in fact of the order of 10^{-3} to 10^{-4}) for a propulsion device to operate over any practical period of time. Since $(\overline{M}_c/M)$ will be of the order of unity, it is obvious that, if this propulsion technique is to be feasible, these devices must operate with values of both α and $\bar{S}$ as small as possible. In fact, since the most optimistic estimates of α predict values of 0.01 (at 1000 v) to 0.045 (at 50 v), it seems clear that operation below $V = 100$ v will be essential in order to achieve acceptably small sputtering ratios, and preferably operation should be limited to the order of 50 v.

Thus low atomic mass elements appear more attractive for any given specific impulse contrary to other considerations of propellant mass discussed earlier which favored larger atomic mass. Hence, one can expect to find an optimum propellant material and, furthermore, an optimum specific impulse for this type of thrustor when one combines performance and lifetime considerations. Viewed from lifetime considerations as well as performance, all of these factors put together indicate that the optimum specific impulse will lie between 2000 and 3000 sec with a corresponding optimum propellant material of the order of 5 to 20 amu (e.g., Li, N_2, Ne, or possibly Na), where the possible use of Li at higher I_{sp} appears especially attractive. Much more experimental development work and evaluation is required, and the choice of cathode material must be made from the viewpoint of minimizing both sputtering and the cathode power requirements.

IV. Electrical Power

It is convenient to express the results in terms of the thrust of the system and to express the power requirements per unit thrust wherever possible. The parameters that have been retained in these expressions are the arc voltage V, the ex-

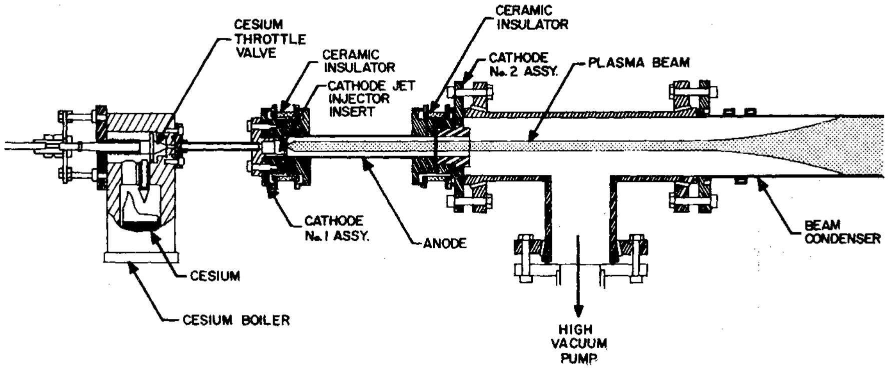

Fig. 2 Cold-cathode plasma generator

haust-arc efficiency η_{ea}, the return-ion parameter α, and the propellant of atomic mass M in amu. It is convenient at times to relate the power per unit thrust to the specific impulse I_{sp} and mass efficiency η_m rather than to V and M. This can be done by noting that $T = M_a I_{sp}$ by definition, and

$$(V/M)^{1/2} = 0.72 \times 10^{-3}(I_{sp}/\eta_m \bar{x}) \tag{29}$$

where $\bar{x}$ is the average of the dimensionless exhaust velocity v/v_0 with $v_0 = (2eV/M)^{1/2}$ expressed as before in the cathode emitter section.

To compute the arc power required per unit thrust, also needed is an expression relating the thrust T to the equivalent current i_+. If one expresses V in volts, i_+ in amperes, M in amu, and T in millipounds-force (1 kgf $\approx$ 2200 mlbf), one finds that the proper numerical factor relating the quantities is

$$i_+ = 30.4 \left[\frac{\bar{x}T}{(VM)^{1/2}}\right] \text{amp} \tag{30}$$

However, since

$$P_{ex} = \bar{y}_e\, i_+ V \tag{31}$$

and the total power $P = P_{ex} + P_{arc}$ required for operation of the device (exclusive of magnets) is

$$P = \bar{y}_e\, i_+ V/\eta_p \tag{32}$$

then

$$\frac{P}{T} = 30.4 \left(\frac{\bar{y}_e \bar{x}}{\eta_p}\right)\left(\frac{V}{M}\right)^{1/2} = 0.022 \left(\frac{\bar{y}_e}{\eta_p \eta_m}\right) I_{sp} \text{ kw/lb} \tag{33}$$

Equation (33), in which the power per unit thrust appears proportional to the specific impulse divided by an appropriate efficiency factor independent of the thrust level, has the form that generally applies to other electrical propulsion systems.

Additional factors contribute to the total power per unit thrust, the major items being magnet power P_M and auxiliary cathode heater power P_c whenever assistance due to thermal emission is required. These power losses, in general, cannot be expressed as simply in terms of I_{sp} and T, and computations of the thermal cathode and magnet power requirements therefore require a detailed parameter evaluation if the results are to be considered at all realistic. The authors have carried out some calculations, and the results indicate that P_c and P_M can be effectively small relative to P_{ex} and P_{arc}.

V. Experimental Program

In a limited experimental program that was undertaken at Electro-Optical Systems, Inc., the objective was to verify and amplify the subjects of the previous discussions. Cesium and argon both were used in a small unsophisticated cold cathode plasma generator[10] shown in Figs. 2 and 3. There are two cathode regions, each very different from the other physically and functionally. The neutral gas fuel is introduced into the arc discharge region as an axially directed jet stream through the center of the closed cathode (cathode no. 1). The second cathode is cylindrically hollow and coaxially oriented with the anode, which is located between the two cathodes. Cathode no. 2 shapes the exhaust plasma, accelerates the ions created in the arc region, and reflects some electrons back into the arc region.

The electrical potential and physical arrangement of the anode and cathodes resemble that of a Penning discharge and apparently would support oscillating electron trajectories in the conventional way. However, the similarity to a Penning source is only partially applicable because 1) the gas jet in the first cathode absorbs so much of the electron energy that oscillations near the central axis are discouraged, and 2) the hollow second cathode supports oscillations only at the extreme radial position.

The injection of fuel through the closed cathode is accomplished through a very small hole (0.035 in. diam) in a cold cathode surface (cathode no. 1) as shown in Figs. 2 and 3. The particular gas jet injector shown is a Prandtl-Meyer flow device[9] but may be any hypervelocity ducted flow as generally described in the field of gas dynamics. This classical flow, of course, pertains only to the gas without ions and without a magnetic field. Only the small high pressure region immediately surrounding the small injector hole is affected where the resultant pressure gradient is localized at the cathode. It is assumed that the secondary electron emission from the cathode is assisted by a fraction of the electrons released through the creation of ions in the immediate vicinity of the cathode, especially in the dense gas region around the jet. This may, in fact, be one of the unique features of the jet, as only in a region of a large gas density gradient plus electric field gradient is it possible to have a fair probability of gas ionization coupled with a significant chance that an electron will completely escape the vicinity. To what extent this process plays a role in the discharge, however, is not known yet. Perhaps, for example, the localized high gas pressure at the cathode in the vicinity of the small cathode holes modifies the potential sheath in such a way that many ions are ejected out of and away from the discharge region.

The experimental plasma generator has been operated continuously for several hours. Magnetic field intensities of 0 to 300 gauss were used in the arc discharge region, and the magnetic field gradient near the exhaust was varied from zero to about 10 gauss/cm. Operation with cesium was ex-

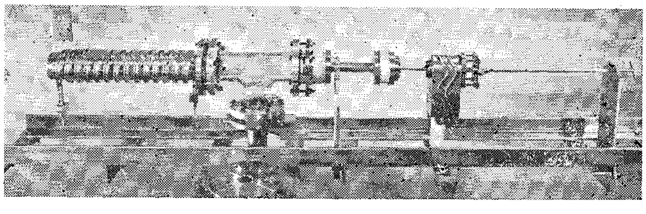

Fig. 3 Cesium plasma generator—side view

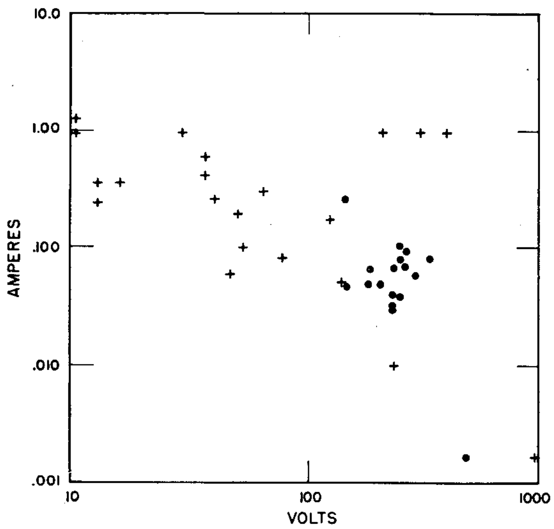

Fig. 4 Cesium arc characteristic

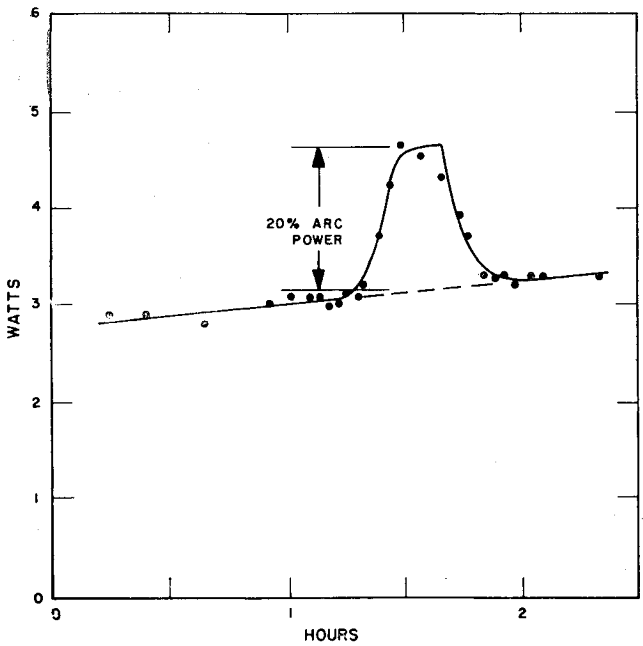

Fig. 5 Cesium beam calorimetry chart

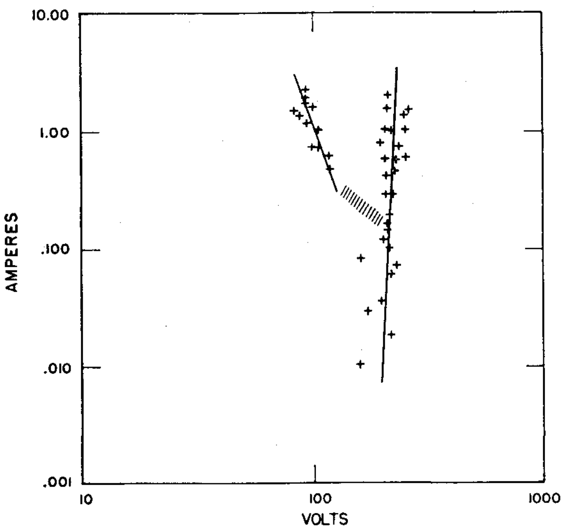

Fig. 6 Argon arc characteristic

tended over about two decades in voltage, and unexpectedly the cold-cathode arc discharge was self-sustained for voltages as low as 10 v. Another surprising and unusual feature was that the current-voltage characteristic was of the negative-resistance type even with relatively low, throttled-down cesium throughput. The volt-ampere curve in Fig. 4 shows a scatter of data and reflects the influence of several operating parameters on the operating characteristic of the cesium arc.

The measured values of power-to-thrust were at first encouraging, but short-lived because the indicated thrust was due to a static electric force acting between the movable and the stationary parts of the pendulous thrust indicator. By grounding the pendulum, shielding against electrostatic effects, and calibrating the electrostatic effect directly with a power supply, it was determined that the thrusts were very small from the cesium arc. Power efficiencies of less than 10% were obtained, and "noise" at the low thrust level of 5 μlb limited a more accurate measurement. It is interesting to note that the "false indications" of the insulated pendulous element corresponded to a "static voltage" essentially equal to that applied to the arc, so that some ions with maximum energy were being collected by the pendulous element.

In a separately conducted calorimetry experiment, the energy input to a beam target was measured. The data for cesium, shown in Fig. 5, were taken under optimistic conditions of zero magnetic field gradient and indicated a maximum power efficiency of 20%.

Operation with argon was radically different from that with cesium. Experimental data were obtained more easily through the use of conventional gas flow methods at room temperature, but the performance was inefficient for propulsion purposes, as was the case with cesium. The argon arc response, shown in Fig. 6, shows two distinct modes of operation where stable operation on either branch depends on pressure, magnetic field intensity, and other parameters.

Table 2 shows typical comparative data for cesium and argon operation where the low values of $\eta_p \times \eta_m$ indicate an order of 1% power in the plasma beam provided that the mass efficiency was unity. The response of the pendulous thrust balance indicator is shown in Fig. 7 and is typical of either cesium or argon except for magnitude.

During the tests using a pendulous thrust indicator, and with different values of magnetic field gradient, it was observed that a thrust of 5 μlb at 300 gauss increased to 75 μlb when only a few tens of gauss were used. Although the operating characteristic of the arc varied due to the change in magnetic field intensity, the large change in the low thrust level is considered as supporting evidence of the difficulty in plasma ejection through a divergent magnetic field as discussed earlier in Sec. II.

VI. Conclusions

In an electric propulsion device, the exhaust plasma must not revert back along magnetic lines to the front of the dis-

Table 2 Typical data for cesium and argon operation

V, v	I_{arc}, amps	B, gauss	T, μlb	P/T, kw/lb	I_{sp},[a] sec	$\eta_p x \eta_m$[b]
Cesium						
1000	0.002	0	<5	>400	3600	<0.10
500	0.003	80	<5	>300	2300	<0.09
200	0.020	170	<5	>800	1600	<0.02
Argon						
260	1.600	0	120	3470	1900	0.0067
230	0.070	0	30	537	1800	0.040
232	0.580	50	30	4500	1800	0.0048
170	0.080	230	30	450	1500	0.040
100	1.600	0	45	3560	1100	0.0037

[a] $I_{sp} = (2eV/M)^{1/2}/g$.
[b] Calculation using Eq. (33), calculated values of I_{sp}, and $\bar{y}_e = 0.6 =$ const.

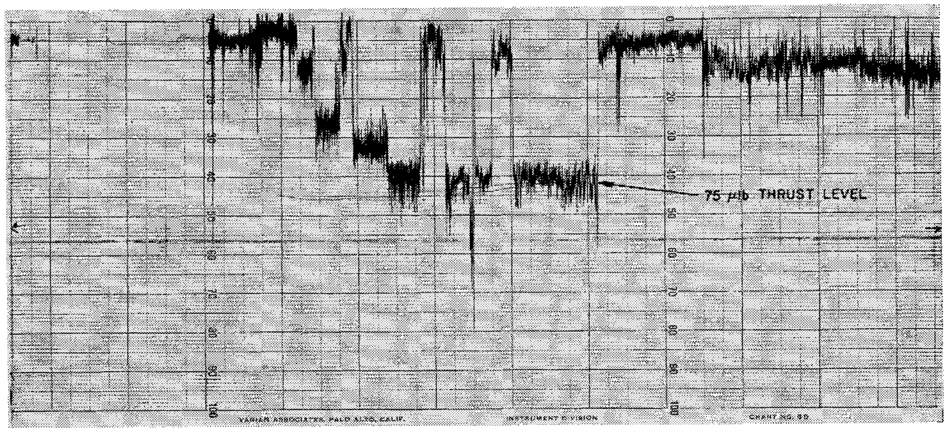

Fig. 7 Thrust collector chart recording

charge tube. Numerical results of an analysis of plasma ejection from the magnetic field have indicated that very small diameter beams and high currents are necessary if the plasma is to escape. Typically, for electron temperatures of 10^4 °K, currents of the order of 100 amps are necessary for a beam of 1-cm radius to leave a magnetic field of a few hundred gauss with a divergence of the order of 5°. Qualitatively, the general effect has been verified experimentally with thrust measurements using a cesium and argon plasma generator.

Cathode sputtering considerations seem to favor the use of a low work function (or thermionic) emitter rather than a conventional cold-cathode secondary emitter because lower arc voltages can be used. However, much more mass could be sputtered from the latter without affecting the operation or modifying the geometrical constants. High secondary electron yields generally come from a high arc voltage and leads to higher sputtering rates except for the alkali metals. However, the choice of cathode material should be made from the viewpoint of minimizing both sputtering and cathode power requirements and optimizing the value of I_{sp}. If a thermionic cathode were to be used, an effective work function of $\varphi \leq 3.5$ ev appears to be necessary in order to keep the cathode heater power within the limits imposed by reasonable operational efficiency requirements.

In considering operation over practical mission times, it appears that there is a maximum feasible operating voltage and specific impulse, and that one of the lighter alkali metals such as lithium or sodium may be an optimum propellant. The use of alkali propellants makes possible the use of a low-voltage cold-cathode configuration for long life operation.

Although a thrust of 75 μlb has been measured at a distance of about a meter away from the exhaust of an experimental plasma generator, the operation was very inefficient with only an order of 1% power in the exhaust beam. When the beam was intercepted by a plate closer to the arc, and therefore in a region of high magnetic field, a calorimetric measurement showed that 20% of the arc power was intercepted. Several different modes of operation of the arc have been observed, especially with operation in argon, and the significance of these data is presently under investigation. However, the difference between operation with cesium and argon is chiefly that the cesium arc voltages are quite low, including cold-cathode operation, so that a great advantage in the use of alkali metals to minimize sputtering was verified.

References

[1] Finkelstein, A. T., "A high efficiency ion source," Rev. Sci. Instr. **11**, 94–97 (1940).

[2] Meyerand, R. G., Salz, F., Lary, E. C., and Walch, A. P., "Electrostatic potential gradients in a non thermal plasma," Proc. Internatl. Conf. Ionization Phenomena Gases, 5th, Munich (1961).

[3] Davis, J. W., Walch, A. P., Meyerand, R. G., and Salz, F., "Charged particle dynamics and electrostatic potential gradients in a neutral plasma," ARS Preprint 2070-61 (October 1961).

[4] Salz, F., Meyerand, R. G., Lary, E. C., and Walch, A. P., "Electrostatic potential gradients in a Penning discharge," Phys. Rev. Letters **6**, 523–525 (1961).

[5] Penning, F. M. and Moubis, J. A., "Neutron tube without pumping device," Physica **4**, 1190–1199 (1937).

[6] Speiser, R. C. and Vernon, R. H., "Cesium ion-atom exchange scattering," ARS Preprint 2068-61 (October 1961).

[7] Spitzer, L., *The Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956).

[8] Grey, D. E., *American Institute of Physics Handbook* (McGraw-Hill Book Co., Inc., New York, 1957), pp. 7–193.

[9] Oswatitsch, K., *Gas Dynamics* (Academic Press, Inc., New York, 1956), p. 259.

[10] Kilpatrick, W. D., "Continuous plasma generator," U. S. Patent 2,892,114 (1959).